A New Species of Argument: The Role of Mathematics in Darwin's Origin of Species

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> I have deeply regretted that I did not proceed far enough at least to understand something of the great and leading principles of mathematics; for men thus endowed seem to have an extra sense. (Charles Darwin, *The Autobiography of Charles Darwin*[1])

> So every idea of Darwin – variation, natural selection, sexual selection, inheritance, prepotency, reversion – seems at once to fit itself to mathematical definition and to demand statistical analysis. (Karl Pearson, *Biometrika*[2])

Charles Darwin gets credit for inspiring the development of concepts, methods, and philosophies in fields as diverse as economics and literature. However, one area in which he is not often commended for being inspirational is in the development of mathematical investigation and argumentation in the study of variation and evolution. The purpose of this essay is to offer arguments and analyses to suggest that in fact Darwin relies heavily on mathematical elements such as quantification and basic arithmetical operations for support and formulation of his arguments for dynamic variation and the principle of divergence of character in the *Origin of Species*. It will also make the case that in choosing to employ these strategies Darwin was attempting to follow the best practices of quantitative induction common to Victorian science and the emerging field of biogeography, which inspired Darwin to develop a robust mathematical science for the study of variation and evolution.

Mathematical Darwin?

Although Karl Pearson, in the quotation above, recognizes a fundamentally mathematical inspiration in Darwin's theories, Darwin scholars in rhetoric and history have either ignored or mostly dismissed the idea that mathematical warrants play a significant role in the *Origin*. In rhetoric, for example, Darwin's persuasive efforts have been the focus of papers by important authors such as John Angus Campbell, Jeanne Fahnestock, Alan Gross, Carolyn Miller and Michael Halloran (Campbell 1974, 1975, 1987, 1989, 2003; Fahnestock 1996; Gross 1990; Miller and Halloran 1993). Their works explore different aspects of his rhetorical strategy in the *Origin* including the use of analogy between the human breeder and nature to help his audience understand the operation of natural selection[3] and the importance of the rhetorical figures incrementum and gradatio in making the argument about variation and diversity among groups of organisms.[4] Despite the wide range of topics and issues covered by rhetorical scholars, no comprehensive discussion about the importance of mathematics in making arguments in the *Origin* has taken place.[5]

Mathematics in *Origin of Species*

The fact that many modern scholars do not believe or have not considered mathematical argument as an important facet of Darwin's persuasive strategy in the *Origin* raises the question: "If mathematics plays such an important role in Darwin's argument, why is it that so few scholars in rhetoric and history bothered to write about it?" Influential historians, notably David Hull, have made the case that Darwin could never have integrated mathematical reasoning into his arguments because this type of reasoning is deductive and cannot be brought into the service of an inductive theory.[9] Like Hull, rhetorical theorists may also have missed the persuasive dimension of mathematical argument in Darwin's work because they assumed that mathematical warrants are deductive. Though this does not necessarily preclude their functioning in Darwin's text, it does remove them, in the minds of many rhetorical analysts, from being the focus of a rhetorical investigation.[10] A careful examination of the disciplinary developments in botany and geology at the beginning of the nineteenth century, a detailed investigation of the development of Darwin's ideas in his notebooks and letters, and a close textual analysis of the arguments in the *Origin*, however, challenge these objections. They reveal that mathematical arguments are essential components in the formulation and defense of Darwin's arguments and that Darwin employed them because he hoped to establish a quantitative science for studying variation and evolution that embraced the nineteenth-century standards for scientific argument.

Keeping Count: The Rise of Statistics in the Nineteenth Century

One of the fundamental characteristics of robust science in both modern and Victorian notions of science was quantification.[11] Without the ability to "translate" natural phenomena into the language of numbers, Victorian scientific philosophers believed that induction leading to the formation of empirical laws could not commence. It was, therefore, the first step in the formation of any science to discover the method or system of measure on which quantitative induction could be founded. Although various attempts had been made in the eighteenth century by Linnaeus and others to quantify certain aspects of biological research, such as the classification of leaves and reproductive organs in plants, they were all considered, at least by nineteenth century standards, artificial and, therefore, not sufficient for the basis of a quantitative science. At the beginning of the nineteenth century, however, two important developments afforded new opportunities for the development of quantitative investigations of organic phenomena. The first was the increased interest in and use of statistics. The second was the discovery of the fossils of extinct organisms whose forms were completely alien from existing flora and fauna, which focused attention on questions about the origin and dispersion of organic forms. In his search of evidence and arguments for the *Origin of Species*, Darwin was influenced by both of these developments, which inspired him to cultivate quantitative evidence and mathematical arguments to support his theories of variation and evolution in the *Origin*.

Though vital statistics (numbers of births and deaths) had been collected since the seventeenth century by religious and political organizations, the number of investigations and degree of attention to their results was limited to very small audiences.[12] At the beginning of the nineteenth century, however, social, political, and economic contingencies converged to create what statistical historian Harald Westergaard dubs the "The Era of Enthusiasm" for statistics and Ian Hacking calls a period with "a professional lust for measurement" (Westergaard 136, Hacking 5). [13] The statistical fever that had affected politicians and moral philosophers in the early decades of the nineteenth century also affected geologists and botanists who were exploring the vast biodiversity of the Americas and Australia. Like the political economists, they began in earnest to gather quantitative data on organic populations and the conditions under which they thrived. However, unlike their counterparts, the ends for their statistical efforts were not primarily for making policy decisions; rather their goals were to explore important scientific questions raised by new geological theories that drastically lengthened the age of the earth and new paleontological evidence of the existence in fossils of organisms that were unlike any flora or fauna known to exist.

These efforts gave rise to a new field of *biogeography*, whose aim was to answer fundamental questions about organic populations, including "What causes influence the thriving or extinction of particular species?" "What is the distribution of species and genera upon the globe?" "What is the population of any given species?" "How can we account for the appearance of new species throughout geological time?" and "What are the laws by which plants and animals of different parts of the earth differ?"[15] One of the assumptions of this new field was that these questions needed to be answered not through classification of organisms and minerals, but rather through the juxtaposition of quantitative facts about climate, population, and location. Alexander von Humboldt, one of the early founders of the field, proclaims this goal in *Aspect of Nature in Different Lands and Different Climates* (1849):

> Terrestrial physics have their numerical element, as has the system of the universe, or celestial physics, and by the united labors of botanical travelers we may expect to arrive gradually at a true knowledge of the laws which determine the geographical and climactic distribution of vegetable forms. (108)

The path toward a new biogeographical physics was forged in works such as Alphonse De Candolle's *Essai Elementaire de Geographie Botanique* [Elementary Essay on Botanical Geography] (1820), Robert Brown's *General Remarks Geographic and Systemic of the Botany of Terra Australis* (1814), and Joseph Hooker's *Botany of the Antarctic Voyage* (1853). In their texts, statistics on temperature, elevation, size of organic populations, and the size and distribution of genera and species were used to make arguments bearing on the questions of distribution, variation, origination, etc.

Robert Brown's work exemplifies the biogeographers' efforts to use basic arithmetical operations and quantified data to compare and make arguments about variation in organic phenomena and the relationship of this variation to geographic and climactic conditions. In *Botany of Terra Australis*, for example, Brown tests the assertion commonly held by nineteenth century botanists that dicotyledonous plants outnumber monocotyledonous[16] plants by examining whether climate affects the numbers of either type in the general botanical population:

> With a view to determine how far the relative propositions of these two classes [dicotyledons and monocotyledons] are influenced by climate, I have examined all the local catalogues or Floras which appear most to be depended on…. The general results of this examination

> are that from the equator to about 30° of latitude, it the northern hemisphere at least the species of dicotyledonous plants are to monocotyledonous plants as about 5 to 1 … and that in the higher latitudes a gradual diminution of dicotyledonous takes place, until at about 60° N. lat. and 55° S. lat. they scarcely equal half their intratropical proportion. (*Miscellaneous Botanical Works* 8)

In the passage, Brown draws on quantitative descriptions of location and previously tabulated statistics on the number of species of each sort of plant as well as calculated ratios to describe the limits of the geographical distribution of dicotyledonous and monocotyledonous plants. He concludes, based on the data and calculations, that dicotyledonous plants are abundant near the equator but become less abundant in northern latitudes. This conclusion provides precise quantitative detail supporting what was otherwise an anecdotal assumption about the difference in dicotyledonous and monocotyledonous in the general plant population. It also supplies new information about the relationship between location and the thriving of dicotyledonous plants that was previously unknown.

Darwin and the Biogeographers

The quantitative data and mathematical methods used by Brown and other biogeographers inspired Darwin to develop quantitative mathematical arguments about the phenomena of variation and evolution. Evidence that Darwin was inspired by their methods can be found in his reading habits, in the private thoughts he recorded in his notebooks, and in his letters discussing the construction of his arguments for the *Origin*. A brief assessment of Darwin's reading habits before and during the development of the *Origin* reveal that Darwin was familiar with the central works of biogeography and read them as he was developing his ideas for his masterwork. He was most likely introduced to them by his mentor John Henslow, who presented him before he left Cambridge, with Humboldt's *Personal Narrative of Travels to the Equinocturnal Regions of the New Continent During the Years 1799-1800*[17] in which the explorer discusses the importance of quantification in the discovery of natural laws, and offers statistics on temperature, altitude, longitude and latitude, etc. for various regions and flora in South and Central America.

During his years of exploration and in the decade following, Darwin continued to read in biogeography. On board the *Beagle*, Darwin had access to a number of works by important biogeographers including Alphonse de Candolle's *Essai Elementaire de Geographie Botanique* and Humboldt's *Personal Narrative*, which he reread. In addition, he received Lyell's *Principles of Geology* volume two in September of 1832 during his voyage. Chapter five of the second volume, "Laws that Regulate the Distribution of the Species," discusses at length all of the important figures of biogeography and the importance of their work to understanding the geographical distribution of plants and animals. When Darwin returned home and began developing the ideas that would be presented in the *Origin*, he continued to show interest in the work of the biogeographers as references to their works and ideas in his notebooks testify. At the end of *Notebook* C in a section titled "To be Read," for example, Darwin lists "Brown at end of Flinders & at end of Congo voyage, Decandolle. Philosophie, or Geographical Distribution. <<in Dict. Sciences. Nat. in Geolog Soc.>>" as texts which he believed would be beneficial to the development of his ideas (C 268).

More strikingly, perhaps, the writings in the *Notebooks* themselves echo the biogeographers' sentiments about the importance of developing a quantitative mathematical approach to the study of plant and animal life. In *Notebook* D, for example, Darwin repeats Humboldt's assertion that investigators of organic phenomena could and should emulate the quantitative inductive methods used in celestial and terrestrial physics:

> Astronomers might formerly have said that God ordered each planet to move in its particular destiny. – In the same manner God orders each animal created with certain form in certain country, but how much more simple, & sublime power let attraction act according to certain laws such are inevitable consequent let animal be

> created, then by fixed laws of generation, such will be their successors. – let the powers of transportal be such and so will be the form of one country to another. – let geological changes go at such a rate, so will be the numbers & distribution of the species!! (B 101-2)

In this passage, Darwin reveals a clear link in his thinking between the methods of astronomy and the study of organic phenomena ascribed to by the biogeographers. Just as astronomers create general laws by quantifying the period of the revolution of celestial bodies, their distance from one another, the amount of space they sweep out in a given period, etc. the biogeographer could arrive at quantitative laws describing the number and distribution of the species by determining the rate of generation of organisms, the rate in change of geological conditions, and the power of transport. Darwin's reading habits and his notebooks supply evidence that he was considering the idea that natural laws might be discovered by working inductively from quantified evidence as he was gathering his thoughts for what would become the *Origin of Species.* However, it was not until the mid-1850s when he began writing the manuscript of *Natural Selection*, his "big species book," that there is evidence that Darwin began to put these ideas into practice by gathering statistical data and making arithmetical calculations to test his theory of evolution and build his arguments.

In 1855 Darwin began lengthy correspondences with Asa Gray and H.C. Watson in which he asked the botanists to supply him with information about genera and species and discussed with them calculations of the ratios of varieties to genera in large and small genera. The quantified data and mathematical calculations discussed in these letters serve as sources of formulation and support for the relationship by descent and the principle of divergence of character in the *Origin*.[18] In his second letter to Asa Gray on 24 August 1855, for example, Darwin hints to Gray that if he could get a reliable systematist to help him identify "close species," i.e. species that closely resemble one another, he could calculate whether there was a propensity for larger genera to have more of these types of species than smaller genera:

> It occurred to me that if I could get some good systematists … to mark (without the object being known) the close species in a list; then if I counted the average number of the species in such genera, & compared it with the general average … of the species to the genera in the same country; it would, to a certain extent, tell whether on average the close species occurred in the larger genera. (Darwin to Gray, 24 August 1855)

Darwin makes a similar request to Watson, who obliges him by marking close species in his catalogues of English plants (Watson to Darwin, 17 August 1855). Using Watson's, Gray's, and other botanists' marked compendiums of species, Darwin searched for proof of a general pattern in the ratios of closely related species in large and small genera. He hoped to find that larger genera have a greater number of close species while smaller genera have fewer more distinct species. He reasoned that if this pattern did exist, it would support his theory of variation and relation by descent because larger, more successful genera, would be producing new species which would be recent and closely related, whereas unsuccessful genera would not be producing newer species and some of the older species which they had produced would have died out, leaving gaps between the existing species making them appear less closely related (Watson to Darwin, 19 November 1854, n. 2).

Origin of Species: a General Overview of the Argument

A cursory examination of Darwin's letters, *Notebooks*, and reading lists reveals that he was familiar with ideas and methods of biogeography and that he adopted them in his search for evidence and arguments to support his theories of variation and evolution. In order to make the case that mathematics is an important aspect of Darwin's rhetorical strategy in the *Origin*, however, it is imperative to show that mathematical argumentation exists in the text and to understand what role it plays in supporting Darwin's conclusions. Toward this end, the next two sections examine the arguments in chapters two and four of the text[19] as well as the peripheral documents associated with them. This investigation reveals that Darwin employs mathematical argument in the book and that this mathematical argumentation plays an important role in helping him formulate and support his arguments for *the existence of a process of dynamic variation* and *the principle of divergence of character*. To understand the significance of the arguments of chapters two and four in the overall scheme of Darwin's argument, it is useful to review the primary conclusions he attempts to establish in the text. The basic arguments are: 1) organisms are highly plastic and can be made to vary to a great degree; 2) variation accumulates over time, resulting in populations of organisms that were once related becoming physically distinct; 3) the spread of variation in nature is the result of natural selection; and 4) the more diversity in a species or genera the more likely it is that its members will successfully reproduce. These primary arguments are introduced and developed in the first four chapters of the book. The other chapters of the text, five through fourteen, are concerned with presenting qualitative evidence from geology, animal behavior, comparative anatomy, and other areas of knowledge that support Darwin's four main arguments as well as his efforts to address possible disputations of his position.

Chapter II: Variation under Nature

In the second chapter of the *Origin*, "Variation under Nature," Darwin argues for the possibility of selection without human intervention through the process of natural selection. He achieves this goal, with the help of quantified comparisons, using arithmetical operations that prove that not only are taxonomic categories of species fuzzy, but also that this fuzziness can be accounted for by conceiving of diversity as the result of the *dynamic process* of continual variation revealing a relationship of descent between the different levels of the taxonomic hierarchy. In the first portion of chapter two, Darwin sets up his argument by refuting the position of special creationists, who believed that each species identified in the taxonomic hierarchy marked a unique creation that was readily identifiable by the existence of an indelible set of features. He argues that if this position is correct, then there should be a definite consensus about which organisms belong in a particular category. In order to test the veracity of this rational assumption, Darwin quantitatively compares the categorization statistics made by experts in the field, including H.C. Watson:

> Compare the several floras of Great Britain, of France, or of the United States, drawn up by different botanists, and see what a surprising number of forms have been ranked by one botanist as a good species, and by another as mere varieties. Mr. H.C. Watson, to whom I lie under deep obligation for assistance of all kinds, has marked for me 182 British plants, which are generally considered as varieties, but which have all been ranked by botanists as species…. Under genera, including the most polymorphic forms, Mr. Babington gives 251 species, whereas Mr. Bentham gives only 112, – a difference of 139 doubtful forms! (41)

Using their own data, Darwin reveals that even experts in plant identification and categorization come to astonishingly little agreement about which organisms should be ranked as varieties and which as separate species. In fact Darwin argues that no classification of species has been formalized. By casting doubt on the fixity of taxonomic categories, he creates an opportunity to present his own theories of evolution and natural selection, which he believes more adequately account for the data.

He opens the second portion of the chapter by clearly laying out for his readers his position:

> Hence I look at individual differences, though of small interest to the systematist, as of high importance for us, as being the first step towards such slight varieties…. And I look at varieties which are in any degree more distinct and permanent, as steps leading to more strongly marked and more permanent varieties, and at these latter, as leading to sub-species and to species…. I attribute the passage of a variety, from a state in which it differs very

> slightly from its parent to one in which it differs more, to the action of natural selection in accumulating … differences of structure in certain definite directions. (44)

In these lines, Darwin presents a vision of diversity in nature as a dynamic process rather than a static condition. He argues that the small differences observed in individual organisms can spread by descent throughout successive generations, making the offspring of those individuals slightly different from the general population from which they originated. These differences can widen through the continued accumulation of variation and eventually transform distinct varieties into distinct species. This dynamic process, Darwin argues, can be attributed to natural selection, which he defines as "the preservation of favorable variations and the rejection of injurious variations" (68). Once he has established his position on the source and character of diversity, Darwin presents arguments with which he attempts to link the size and range of a group of organisms at a particular level of the taxonomic hierarchy to the number of subordinate categories of organisms associated with that group. In order to do this, he depends both on arithmetical calculations using quantitative data, and on the rhetorical-logical commonplace [*topos*] of the *more and the less*, with which he establishes a connection between size and range of a population and the characteristic of diversity.

As a student of religion and classics at Cambridge, it is likely that Darwin would have been either directly or indirectly introduced to Aristotle's *topoi* in the *Topics*. If so, he would have likely encountered the classical commonplace rule of arguing from the *more and the less*, which suggests that you can make the case for a link between a subject and an accidental phenomenon if you can establish that when the subject increases or decreases, the accidental predicate also increases or decreases. In Book II of the *Topics* Aristotle explains that

> Moreover, argue from greater and less degrees. There are four commonplace rules. One is: see whether a greater degree of the predicate follows a greater degree of the subject…. For if an increase of the accident fol-

> lows an increase of the subject, as we have said, clearly the accident belongs; while if it does not follow, the accident does not belong. You should establish this by induction. (114[b] 35-115[a] 6)

Using the statistics from available botanical compendia, Darwin calculates the number of botanical varieties belonging to species with the greatest estimated population sizes and ranges in hopes of discovering some general pattern in or connection between these species: "I thought that some interesting results might be obtained in regard to the nature and relations of the species which vary most, by tabulating all the varieties in several well-worked floras" (Darwin, *Origin*, 45). To make his calculations Darwin first divides the species in the compendia into large and small species according to the author's size designations. He then divides the number of species in the large and small groups by the number of varieties that are connected with them to produce an average of the number of varieties for each species for large and small species.[20] The results of these tabulations and comparisons reveal that there is a correlation between the size and range of a species' population and the number of varieties recorded for that species:

> In any limited country, the species which are most common, that is abound most in individuals, and the species which are most widely diffused within their own country … often give rise to varieties sufficiently well-marked to have been recorded in botanical works. Hence it is the most flourishing, or, as they may be called, the dominant species … which oftenest produce well-marked varieties. (45-46)

As the result of his calculations, Darwin discovers, or confirms, that the more populous species tend to have a greater number of identified varieties associated with them. This correlation is accounted for by his dynamic theory of natural diversity because statistically a correlation between the size of a population and the development of sub-populations would be expected as larger populations would have more offspring and, therefore, a greater number of variations to be selected.

The correlation between the calculated size of a species and the number of varieties associated with it supports Darwin's argument that diversity in nature is the result of the production of variations. In order to strengthen the conviction of the audience that this relationship is legitimate and to make the case that the relationship exists at all levels of the taxonomic hierarchy, Darwin predicts that the same relationship will be found between genera and species. To validate this prediction, he conducts further calculations and comparisons to assess whether or not the principle holds true at the taxonomic level of genera:

> If the plants inhabiting a country and described in any Flora be divided into two equal masses, all those in the larger genera being placed on one side, and all those in the smaller genera on the other side, a somewhat larger number of the very common and much diffused or dominant species will be found on the side of the larger genera. (46)

Here, Darwin affirms his quantitative prediction that the same correlation between population size and variation which exists between species and varieties also exists between genera and species. Strengthened by the predictive power of his model and the accumulating evidence, Darwin makes a point to emphasize the success of his theory in accounting for the patterns revealed by his calculations:

> From looking at species as only strongly-marked and well-defined varieties, I was led to anticipate that the species of the larger genera in each country would oftener present varieties, than the species of the smaller genera; for wherever many closely related species … have been formed, many varieties or insipient species ought, as a general rule, to be now forming…. On the other hand, if we look at each species as a special act of creation, there is no apparent reason why more varieties should occur in a group having many species, than in one having few. (46-47)

In addition to playing up the success of his prediction, Darwin also challenges the special creationist position to account for the same results. If the different taxonomic categories did in fact represent unique populations of organisms that shared no relationship with other populations, then what would account for the correlations which his comparisons reveal? Though opponents of his theory might argue that these correlations are random coincidence, Darwin suggests here that the fit between the patterns he describes in the quantitative data and the process of variation that he proposes in his theory is too good to be coincidental (47). An analysis of the arguments in the second chapter of the *Origin* reveals that Darwin employed quantitative comparison and basic arithmetical operations to support his theories of dynamic variation and relation by descent between the different levels of the taxonomic hierarchy. In the opening portion of the chapter, he uses quantitative comparison with precise numerical values to challenge the veracity of the existing paradigm of special creation by revealing that among experts there is no clear consensus on the categorization of organisms in nature.

Once he has cast doubt on the theory of his opponents and offered his own, he accumulates evidence to support it. With evidence from calculated averages and precise numerical comparison and argument from the commonplace of the *more and the less*, he reasons that a correlation exist between the size of populations and the development of recognized variations within related sub-populations. These connections suggest that there is a relationship of descent between different levels of the taxonomic hierarchy wherein varieties associated with a given species are actually variations of a common ancestor and so forth up the taxonomic hierarchy. Without the support of overt quantitative comparisons and behind the scenes mathematical operations, Darwin's argument for the existence of relation by descent would have been purely speculative. But by using precise data and mathematical comparison and a quantitatively informed commonplace, Darwin hoped to establish an *ethos of precision and rigor* commensurate with the practices of more robust scientific argumentation that appealed to his target audience of geologists, botanists, and zoologists,[21] who like himself were caught up in the biogeographical revolution.

Chapter IV: Natural Selection and Calculating Diversity

With the evidence and arguments in place that diversity in nature is the result of the spread of variations through organic populations and that a struggle for existence takes place in nature,[22] Darwin proceeds to describe the details of species formation by natural selection. In the fourth chapter of the *Origin of Species*, arithmetical computation and comparison of ratios helps Darwin to discover and support new lines of argument about selection and species formation: 1) the more diversified a group of organisms, the better they will do in their struggle for existence; and 2) the success of highly divergent organisms in part explains how great degrees of difference come to exist between related species. In the initial stages of his calculations of the ratios of varieties to species, Darwin divided the total number of organisms he was investigating into "large" and "small" groups and calculated the average number of varieties for each species and species for each genus in these size categories.[23] He then compared the average number of varieties calculated for the large and small categories of species and genera to determine whether there was a correlation between the size of a species or genera and the amount of variation produced. This argument strategy is explained in the detailed discussion of chapter two of the *Origin*. These calculations revealed that those genera and species designated "large" had more species and varieties, which supported Darwin's conclusion that there was a relationship between the size and range of a population and the number of variations it had. A communication from Sir John Lubbock, the son of Darwin's neighbor at Down in the early summer of 1857, however, appraised Darwin that building his case on assumed average estimates of size created problems in establishing rationally defensible comparisons of the relative degree to which larger genera might be better producers of species and varieties. Lubbock suggested that instead of *averages* Darwin should calculate the *ratios* of varieties to species in large genera and then use them to predict the expected ratio of varieties to species in small genera. Although Darwin was initially skeptical about Lubbock's suggestion, he reworked his estimates with good results.

> I have divided the New Zealand Flora as you suggested, there are 329[24] species in genera of 4 [species] and upwards, and 323 in genera of 3 [species] and less. The 339 species have 51 species presenting one or more varieties. The 323 species have only 37. Proportionately (339 : 323 :: 51 : 48.5) they ought to have had 48 1/2 species presenting vars. So that the case goes as I want it, but not strong enough, without it be general, for me to have much confidence in. I am quite convinced yours is the right way; I had thought of it, but should never have done it had it not been for my most fortunate conversation with you. (Francis Darwin, *More Letters*, 104)

In working out the projected ratios of varieties to species in large and small genera, Darwin discovered that in the case of small genera there are fewer varieties than expected or in the case of larger genera more varieties than expected. The result of these calculations shifted Darwin's attention away from the correlation between the size and range of a group of organisms at a particular level of the taxonomic hierarchy and the number of subordinate categories of organisms associated with that group and towards the importance of variety to the evolutionary success of organic populations. In a letter to Hooker in August of 1857, Darwin's belief in the importance of these calculations to the development of his theory is evident:

> I intend dividing the varieties into two classes, as Asa Gray and Henslow give the materials, and, further, A. Gray and H.C. Watson have marked for me the forms, which they consider real species, but yet are very close to others; and it will be curious to compare results. If it will all hold good it is very important for me; for it explains, as I think, all classification, *i.e.* the quasi-

> branching and sub-branching of forms, as if from one root, big genera increasing and splitting up, etc., as you will perceive. But then comes in, also, what I call a principle of divergence, which I think I can explain. (Darwin to Hooker, 22 August 1857)

This shift of attention toward the importance of the breadth of variation to evolutionary success inspired Darwin to think more carefully about why variation might matter so much in the process of selection. These investigations also lead him to develop his *principle of divergence of character*, which responds to a question he and his critics considered a major obstacle to any theory of variation: "How do the small differences that are observable between populations of closely related species and varieties grow into the large differences that we see between genera, families, etc.?" (Browne, "Botanical Arithmetic," 74). Because the calculated ratios showed that greater variety was a hallmark of species and genera with larger populations and ranges, Darwin felt encouraged to explain evolutionary success in terms of variation. He reasoned that when a species reaches equilibrium between its numbers and resources the only way it could continue to grow is through diversification. Evidence from the *Origin* suggests this line of thinking:

> Take the case of a carnivorous quadruped, of which the number that can be supported in any country has long ago arrived at its full average. If its natural powers of increase be allowed to act, it can succeed in increasing … only by its varying descendants seizing on places at the present occupied by other animals: some of them, for instance, being able to feed on new kinds of prey … some inhabiting new stations…. The more diversified in habits and structure the descendents of our carnivorous animal became, the more places they would be enabled to occupy. (93)

Using the hypothetical case of the carnivorous quadruped, Darwin argues that a species' survival and replication can be improved if some of its members can expand, through variation, into ecological niches where there is less competition for resources with other conspecifics. In a separate passage, he extends this reasoning and concludes that because larger genera and species produce greater degrees of variation, they will be better off in the struggle for existence:

> In a large genus it is probable that more than one species would vary…. In each [large] genus, the species, which are already extremely different in character, will generally tend to produce the greatest number of modified descendents; for these will have the best chance of filling new and widely different places in the polity of nature. (99)

In addition to making the case that bigger is better, Darwin argues by implication here that the breadth of diversity of larger genera and species promotes further diversity and population expansion. This means that larger genera and species will most likely continue to grow and outstrip their smaller counterparts unless checked by unforeseen circumstances. The supposed correlation between size and variation assumed in this passage suggests that for Darwin bigger genera and species are bigger in the first place *because* they are more diverse. Thus, diversity itself becomes a crucial factor in evolutionary success. Working from the assumption that diversity is a primary factor in selection, Darwin proposes to answer the question, "How do small differences between populations of closely related species and varieties grow into large differences?" with his *principle of divergence of character*. He presents this principle somewhat inconspicuously in a breeding analogy at the beginning of the section "divergence of character" about half way through chapter four:

> A fancier is struck by a pigeon having a slightly shorter beak; another fancier is struck by a pigeon having a rather long beak; and on the acknowledged principle that 'fanciers do not and will not admire a medium standard, but like extremes,' they both go on … choosing and breeding birds with longer and longer beaks, or with shorter and shorter beaks. (92)

In chapter four, this analogy acts as jumping off point for launching the reader into the hypothetical "natural" example of the carnivorous quadruped discussed earlier. The "fancier" who selects which features to propagate becomes a conceptual ground for "nature." The large and small beak sizes and the "acknowledged principle" that "fanciers do not admire a medium standard" prepare the reader to understand and accept the importance of variation as a principle of selection. Despite these similarities, one important aspect of the analogy, the fancier's continuous choice of the shorter and longer beaks, is not carried over. This small exception deserves attention because with it Darwin makes a case for his *principle of divergence of character*. The principle is as follows: if we assume that variation is a primary factor in evolutionary success, and if we assume that for each generation the most varied individuals will be selected – i.e., the "fancier" aka "nature" will consistently avoid the "medium standard" – then it must be the case that over time, qualities like beak size, which begin relatively close to each other, will be steadily pushed to extremes until members that were once part of the same population are so transformed that they will be perceived as completely different types of organisms. Thus, the answer to the question, "How do small differences between populations of closely related species and varieties grow into the large differences?" is that small differences get magnified into larger ones because selection favors variety over similarity.

An examination of the fourth chapter of the *Origin of Species* as well as peripheral textual materials reveal the importance of ratios in focusing Darwin's attention on the centrality of variation to evolutionary success and in supplying him with the inspiration for his *principle of divergence of character*. By working out the ratios for the number of species that might be expected in a large-sized genus and the number of varieties that might be expected for a given species, Darwin discovers that the actual ratio of variations in smaller groups of organisms are smaller than the predicted ratios. This discrepancy encourages Darwin to focus his attention on variation and to conclude that degree of variation is an important selection factor. Further, it leads him to argue that continual selection of highly variable organisms explains how related organic forms can become so radically different that they become different species or break off over time to form separate genera (Darwin, *Origin of Species* 100-101*)*.

The Success and Failures of Darwin's Mathematical Arguments with His Audience

A close textual examination of the arguments in the *Origin* suggests that Darwin used quantification and arithmetical calculation to formulate and support some of his most important conclusions. It remains to be determined, however, whether using mathematical arguments had any positive effect on his audience of naturalists' opinions about the validity of these conclusions. This evaluation is particularly difficult given that the *Origin* was only an abstract of Darwin's arguments and, therefore, lacked the full articulation of the evidence and arguments presented in the manuscript for *Natural Selection*. To get a sense of the effect of the mathematical argument in both of its manifestations, I will examine both comments and reviews by audience members who were aware of the full scale of mathematical evidence and those who were not.

Because the manuscript of *Natural Selection*, which contained Darwin's data and calculations, had to be trimmed for publication, the *Origin* included only a few references to his mathematical reasoning. Consequently, readers and reviewers of the text, unless they were extremely diligent in their probing of Darwin's arguments, were not aware of the mass of quantified data and numerous arithmetical calculations which supported his arguments. Many not-in-the-know reviewers who commented on Darwin's method seemed to support John Herschel's view that Darwin's theory of evolution and natural selection were "the laws of higgledy-piggledy"[25] because there was no clear inductive path from experiment or observation to the conclusions Darwin had established. This point is taken up at length, for example, by British mathematician and geologist William Hopkins in *Fraser's Magazine*. After describing at length the well-established steps of induction, he analyzes Darwin's arguments based on their principles. In his closing statements, Hopkins concludes obliquely that the main fault of Darwin's reasoning is that he begins with the statement of larger laws without the necessary proof that would lead up to them inductively. He suggests that Darwin, like Newton, is trying to construct grand theories instead of being a gatherer of evidence, like Kepler, whose data was necessary for the development of Newton's conclusions. In a piece of advice to other natural philosophers eagerly searching for the truth about nature, he warns them not to fall prey to the delusions of grandeur that affect Darwin:

> Let him [the earnest lover of nature's truth] pursue his researches recollecting that biological science requires at present its Kepler's rather than its Newton's – the discovery of the more obvious laws according to which its phenomena may be arranged, rather than its attempts at that higher generalization which may account for those laws by the operation of physical causes. (90)

A similar, less oblique criticism of Darwin's method is leveled by the renowned geologist Adam Sedgwick in the *Spectator*. In his review of the *Origin*, Sedgwick challenges the validity of Darwin's claims on the ground that they have not been drawn inductively:

> I must in the first place observe that Darwin's theory is not *inductive*, – not based on a series of acknowledged facts pointing to a *general conclusion*, – not a proposition evolved out of the facts, logically, and, of course including them. To use an old figure, I look on the theory as a vast pyramid resting on its apex and that apex a mathematical point. (Qtd. in Hull 160)

In the final line, Sedgwick's description of Darwin's theory as a vast pyramid resting on its apex represents a proper epistemological system turned on its head. Instead of the moving inductively from a large base of reliable empirical evidence towards a theory describing that evidence, Darwin, in Sedgwick's estimation, had begun with theory and left the evidence to be discovered by others. Curiously, however, instead of referring to the top of the pyramid as deductive natural law, he refers to it as having "a mathematical point." This would make sense on the grounds that in Victorian science—where Newton was hailed as the paragon of scientific achievement—a mathematical description of natural phenomena was considered the apex of scientific effort. However, it seems odd in a critique of Darwin's work to accuse him of resting his work on a mathematical point when in fact the mathematical reasoning that he has used in the text was so obscure and grounded in empirical statistical data. In fact, the mathematical quality of Darwin's argument is neither mentioned again in Sedgwick's critique in the *Spectator* nor does it appear when Sedgwick makes the same critique elsewhere.[26] Although, it remains something of a mystery as to why Sedgwick would use the phrase a "mathematical point," it seems likely that this was a general reference to the fact that mathematical laws had a privileged position at the top of the Victorian pyramid of knowledge rather than a critique of Darwin's work as resting too much on mathematical reasoning.

Though it is impossible to say whether the publication of the complete manuscript of Darwin's *Natural Selection* would have made his conclusions more palatable to Hopkins and Sedgwick, members of his audience such as Watson, Gray, and Hooker who had access to the quantitative evidence and were sympathetic to the goals of biogeography were more amenable to his conclusions. In a letter from Watson on 8 November 1885, for example, Watson describes his feelings about the legitimacy of Darwin's method of statistical argument as well as his reservations about the conclusions Darwin draws from those arguments:

> You may correctly believe that I *do* take an interest in your investigations about the numerical evidences or indications of variability. There seems to be this difference between us, that you regard the proportions as approximating more towards evidences of a general law, while I see them only as fainter indications of it. (H.C. Watson to Darwin, 8 November 1855)

In these lines, Watson offers support for Darwin's method of using quantitative evidence and arithmetical calculations to make his case, but complains that Darwin has been too quick to make grand claims from his data. However, unlike Hopkins and Sedgwick he is not ready to dismiss Darwin's conclusions as without merit or evidence. Instead, he believes they are credible but at a lower level of certainty than he believes Darwin is intending.

Like Watson, Gray seems to support Darwin's mathematical approach; however, unlike Watson, he seems to be more convinced that Darwin's conclusions are supported by his evidence and calculations. In his lengthy review of the *Origin* in *American Journal of Science and Arts* (1860), Gray clearly states his acceptance of Darwin's quantitative arithmetical argument that the correlation between the size and range of a genus' population and the number of species associated with it is evidence of a relation of descent:

> That the most flourishing and dominant species of the larger genera on an average vary most (a proposition which can be substantiated only by extensive comparisons, the details of which are not given); and … that in large species the genera are apt to be closely but unequally allied together…. The fact of such association is undeniable; and the use which Darwin makes of it seems fair and natural. (170)

Of particular interest in this section of Gray's review is the comment in the parentheses in which Gray alludes to the extensive quantitative comparisons that he knows Darwin has undertaken to make his case. By alluding to them in his review he assures his readers that Darwin has done his due diligence in gathering and comparing data to support his conclusions and that the reviewer has, based on his knowledge of Darwin's arithmetical calculations, found these proofs compelling.

Like Gray, Hooker also supports Darwin's conclusions and his mathematical approach to developing his arguments in the *Origin*. Hooker voices his belief in his introduction to volume three of *The Botany of the Antarctic Voyage the Flora of Tasmania* (1860) which is previewed in the *American Journal of Science and Arts*. In this introduction, Hooker admits to his own stunning conversion by Darwin and Wallace's theory from his previous belief that species were fixed:

> In the introductory essay to the *New Zealand Flora*, I advanced certain general propositions as to the origin of species … amongst others was the still prevalent doctrine that these are … created as such, and are immutable. In the present essay I shall advance the opposite hypothesis, that species are derivative and mutable; and this chiefly because … every candid mind must admit that the facts on which he has grounded his convictions require revision since the recent publication … of the ingenious and original reasonings and theories of Mr. Darwin and Mr. Wallace. (2)

Some of the "ingenious and original reasoning" that persuaded Hooker came from Darwin's arithmetical calculations with quantified data. This fact is evidenced by a statement later on in the introductory essay where Hooker states his agreement with Darwin's principle of divergence of character, the calculations of which Darwin communicated to Hooker in August 1857: "Mr. Darwin addresses another principle in action amongst living organisms as playing an important part in the origin of species, viz., that the same spot will support most life when peopled with very diverse forms" (15).

Though Hooker supported Darwin's reasoning, he was skeptical that Darwin's calculations were reliable because they depended on distinctions between varieties and species that had no natural basis. He voiced his concern in a letter to Darwin in March 1858:

> Botanists do not attach that *definite* importance to varieties that you suppose, they do not treat large & small genera equally & similarly, & the sum of inequalities thus produced tends to make the species of small genera look more invariable than of big…. So my dear Darwin do not be in a hurry with your conclusions. (Hooker to Darwin, 14 March 1858)

In these lines, Hooker warns Darwin that quantitative argument is robust only to the degree to which the mathematical evidence can be linked to natural reality. Despite their misgivings, Hooker, Watson, and Gray – all well-respected botanists – accepted Darwin's arguments and in some cases completely reversed their views on the origin of species. That these experts were persuaded is a testament to the strength of Darwin's strategy to make his case for dynamic variation and the principle of divergence of character using quantified evidence and arithmetical calculations.

Conclusion

With the quantitative data and arithmetical calculations hidden, it is no surprise that historians and rhetoricians of science, like some of Darwin's contemporaries, found no evidence of mathematical argumentation in Darwin's work. However, to say that Darwin could not have used mathematics to make his arguments ignores the evidence in the letters, notebooks, manuscripts, and the text of the *Origin* that Darwin was attempting to uncover laws of nature through the best practices of quantitative induction. A close examination of his reading habits and notebooks reveals that Darwin was dedicated from very early in his scientific endeavors to developing a mathematical approach to the study of biological phenomena. His communications with Watson, Gray, and Hooker as well as in the construction of his arguments in the manuscript of *Natural Selection* give evidence that he put these beliefs into practice. In the construction of his arguments, Darwin was also trying to follow the best practices of making scientific knowledge. Though he offered no formulas for describing selection or the principle of divergence of character, he did depend on quantified data to make his case, and he did attempt to use these data to discover or supply evidence for the existence of patterns in natural phenomena using arithmetical calculations.

Given the controversial nature of Darwin's arguments and the considerable authority that the quantitative inductive method had with Victorian scientists and biogeographers, it is likely that Darwin's efforts to quantify his data and reach conclusions using arithmetical arguments was not only an effort to practice science according to the highest standards, but also to persuade his audience that his radical conclusions were correct because they were arrived at by a process that strictly adhered to proper inductive method. Ironically, despite all of his best efforts to construct rigorous quantitative hypotheses of variation and the principle of divergence of character, Darwin's ideas only "seem" mathematical to Pearson and appear to have no quantitative basis to others because of an unexpected turn of events. With the emergence of Wallace's paper, Darwin was forced to abandon his plans to publish a detailed discussion of his arguments and evidence. Instead, he put out an abstract of his work. In the process of summarization, much of the quantitative evidence and mathematical calculations supporting his theory were left out. As a consequence, Darwin's work was vulnerable to the charges of the methodological weakness that he had assiduously attempted to avoid by using quantified evidence and arithmetical calculations to confirm and formulate his hypotheses about variation and the divergence of character.

Through a close rhetorical examination of Darwin's arguments as well as a careful assessment of his reading habits, ideas, and correspondences, it is possible to recover Darwin's efforts to construct robust quantitative theories of dynamic variation and divergence of character in accordance with the principles espoused by Victorian science and the emerging field of biogeography. These efforts suggest that perhaps more credit needs to be given to Darwin as a pioneer of mathematical argument in the study of organic phenomena whose every idea "fit itself to mathematical definition" and demanded statistical analysis. In addition, they recommend the importance for rhetoricians to pay attention to the mathematical dimensions of argument, the neglect of which has until now obscured important aspects of Darwin's arguments and their reception.

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Notes

Portions of this essay originally appeared in *Rhetorica* 27.1 (2009): 76-97, by the International Society for the History of Rhetoric and the Uni-

versity of California Press.

[1] Charles Darwin, *The Autobiography of Charles Darwin 1809-1882 with The Original Omissions Restored Edited and with Appendix and Notes by his Granddaughter Nora Barlow*, ed. Nora Barlow (St. James Place, London: Collins, 1958), 58.

[2] Karl Pearson, "The Spirit of Biometrika," *Biometrika* 1.1 (1901) 3-4.

[3] Jon Angus Campbell, "Charles Darwin Rhetorician of Science," *The Rhetoric of Human Sciences Language and Argument in Scholarship and Public Affairs*, ed. John S. Nelson (U of Wisconsin P, 1987), 69-86.

[4] Jeanne Fahnestock, "Series Reasoning in Scientific Argument: Incrementum and Gradatio and the Case of Darwin," *Rhetoric Society Quarterly* 26.4 (1996) 13-40.

[5] Though historians of science have expended considerably more effort tracing the development of different mathematical fields and examining their political, cultural, and even rhetorical influence,[6] they also have not, with a few rare exceptions, taken up investigations into the role of mathematics in Darwin's arguments. A survey of 11 books and nine articles, most of which were published in the last 25 years, reveals that few texts associate Darwin's arguments with mathematical reasoning.[7] Those that do (Ghislen 1969, Hull 1973, Gale 1982, and Depew and Weber 1995) predominantly comment either on the lack of mathematical reasoning in the text or on Darwin's inability to use mathematics to make his case. Only four, (Browne 1980, Schweber 1980, Parshal 1982, Bowler 1990), assign any real importance to mathematics in Darwin's arguments.[8]

[6] Silvana Patriarca, *Numbers and Nationhood: Writing Statistics in Nineteenth Century Italy* (Cambridge UP, 1996) and M.J. Cullen, *The Statistical Movement in Early Victorian Britain* (Hassocks, Sussex: Harvester, 1975) are examples of these types of texts.

[7] See Appendix A for the complete list of books and journals used in this survey.

[8] All told, out of 26 texts specifically devoted to discussing the logical contents or development of Darwin's argument, only four clearly

suggest that mathematics has any significant role to play (Browne 1980, Schweber 1980, Parshall 1982, and Bowler 1990). That works out to about roughly 15% of texts recognizing the importance of the role of mathematics to Darwin's argument, 15% dismissing it, and 70% ignoring it altogether.

[9] In *Darwin and His Critics*, Hull takes the position that because Darwin was developing arguments in biology, deductive mathematical reasoning could not aid him in prosecuting his argument:

> Darwin could not help but know the crucial role which mathematics had played in physics, since Herschel had repeatedly emphasized it in his *Discourse*, but it did not seem to be in the least useful in his own work in biology…. For Darwin, mathematics consisted of deductive reasoning, and he distrusted greatly "deductive reasoning in the mixed sciences." In his own work, he seldom was presented with a situation in which he could use such deductive reasoning. (12-13)

[10] While to my knowledge there has been no explicit statement such as Hull's that this consideration has kept rhetoricians from examining the mathematical aspects of Darwin's arguments, this restriction abounds in influential theoretical texts such as Chaim Perelman and Lucie Olbrechts-Tyteca's *The New Rhetoric*. In the very first chapter of the text, the author's set out the definition of rhetoric in contradistinction to the reasoning of mathematical logic:

> Our procedure will differ radically from that adopted by those philosophers who endeavor to reduce reasoning … by taking their cue from the models provided by the deductive or experimental sciences…. Quite the opposite: we will draw our inspiration from the logicians but only to imitate the methods which they have used so successfully. (10)

[11] Quantification and its importance is talked about at length in the leading philosophies of science in the early and mid-nineteenth century:

John Herschel's *A Preliminary Discourse on the Study of Natural Philosophy* (1830) (321) and William Whewell's *Philosophy of the Inductive Sciences* (1840) (1:49).

[12] Life insurance companies, which emerged in the middle of the eighteenth-century, were particularly interested in vital statistics. They used them to produce life tables for calculating annuities (Cullen, 7-8).

[13] There is no single agreed-upon cause for this sudden interest in and collection of statistics. Some historians attribute it to the need for precise measurement required by the Industrial Revolution, which had gathered considerable momentum by the beginning of the nineteenth century (Hacking 5). Others argue that it was the result of a sudden increase in the availability of statistical information that followed the end of the Napoleonic wars (Chatterjee 267). Still others contend, perhaps most convincingly, that the supply of statistical data increased to meet a greater demand by governments who required quantitative data in order to make better-informed political decisions and more persuasive policy arguments (Westergaard 1968, Cullen 1975, Patriarca 1996).[14] In particular, governments required statistics on birth and death rates as well as the resources of their domain and the domain of other nations to make rational economic policy decisions.

[14] Westergaard, 1968, 141; Cullen, 1975, 19-20; Patriarca, 1996, 13-14.

[15] This final question is a direct quote from William Whewell's *History of the Inductive Sciences*, 475.

[16] Monocotyledons and dicotyledons are the two major classes of flowering plants. Monocotyledons are so named because they have a single cotyledon, or embryonic leaf, in their seeds whereas dicotyledons have two. ("Monocotyledon" Wikipedia Screen one accessed July 22, 2008).

[17] Silvan Schweber, "Darwin and the Political Economists: Divergence of Character," *Journal of the History of Biology* 13.2 (1980), 205 n. 25.

[18] A complete accounting of Darwin's argument is given in the section, "Variation under Nature."

[19] Only chapters two and four are examined here for the sake of brevity. I have done a detailed analysis of the arguments in each of the first four chapters and found in all of them that Darwin uses mathematical argument to support his key ideas.

[20] See "Appendix B" for Darwin's mathematical calculations.

[21] In a memorandum on page 94 of the *Natural Selection* manuscript, there is a note in which Darwin identifies his primary audience as "geologists and zoologists" (Charles Darwin, *Charles Darwin's Natural Selection*, ed. R.C. Stauffer [Cambridge UP, 1975], 94. Given that many of the examples and arguments in the text are drawn from botany, I am also assuming that Darwin considered botanists to be a primary audience.

[22] Arguments for natural selection and the struggle for existence are made in Chapter 3 of Origin, "Struggle for Existence."

[23] See "Appendix C" for Darwin's mathematical calculations.

[24] This figure should be "339" but Darwin erroneously writes "329" in his letter. I am assuming that the former is the correct figure because it appears twice as the value for the number of species in genera of four and upwards.

[25] Darwin explains to Lyell that this is how Herschel's reaction to the law had been described (Darwin to Lyell, 10 December 1859).

[26] In a reply to the philosopher John Stuart Mill, Sedgwick wrote that Darwin's theory in the *Origin* "is like a pyramid poised on its apex. It is a system embracing all living nature … yet contradicting point blank the vast treasure of facts that the Author of Nature has, during the past two or three thousand years, revealed to our senses" (Qtd. in Hull 169).

Appendix A

List of Scholarly Books Analyzed for Citations of the Role of Mathematics in Darwin's Arguments.

Aydon Cyril. *Charles Darwin: The Naturalist Who Started a Scientific Revolution*. New York: Carroll and Graf Publishers, 2002. Aydon doesn't mention mathematics in the index of his text. His only reference to it is in his comment that Darwin was not particularly good in mathematics in school on p. 30.

Bowlby, John. *Charles Darwin a New Life*. New York: Norton, 1991. Bowlby doesn't mention mathematics in the index of his text. Chapter 22 "My Abominable Volume" discusses generally the contents of the argument in Darwin's text. It does not, however, go into specifics. Also, it does not include any mention of mathematics.

Bowler, Peter. *Charles Darwin the Man and his Influence*. Cambridge: Cambridge UP, 1990. Bowler doesn't mention mathematics in the index of his text. On P104 of his text he mentions that Darwin undertakes arithmetical studies of the number of species to prove that population pressure within a given area forces species to diversify in order to compete more successfully. On P 117 he also discusses the importance of mathematical reasoning in Malthus' argument which informs his concept of natural selection.

Brent, Peter. *Charles Darwin a Man of Enlarged Curiosity*. New York: Harper and Row, 1981. Brent doesn't mention mathematics in the index of his text. In skimming chapters 4 and 5 "Origin of all My Views" and "The Speculatist" I found no discussion of mathematics.

Browne, Janet. *Charles Darwin the Power of Place*. Vol. 2. Princeton: Princeton UP, 1996. Browne doesn't mention mathematics in the index of her text. She talks at length in chapter 2 "My Abominable Volume" about Darwin's use of evidence but at no time discusses the role of mathematic notation in presenting that evidence.

Campbell, John Angus. "Charles Darwin Rhetorician of Science." *The Rhetoric of the Human Sciences Language and Argument in Scholarship and Public Affairs*. Ed. John S. Nelson. Madison: The U of Wisconsin P, 1987. 69-86. Campbell does not discuss the role of mathematics in his chapter of the text.

Gale, Berry G. *Evolution without Evidence Charles Darwin and the Origin of Species*. Albuquerque: U of New Mexico P, 1982. "Mathematics" in the index: p. 13-14 discusses Darwin's weakness in mathematics. Page 79 explains that Darwin read *Nature* but that he did not understand most of the articles which used mathematics. / Gale discusses the contents of Darwin's arguments in depth. He looks at his arguments, their weaknesses and the problems of evidence in chapter 7 of the text. At no point does he discuss the role of mathematics in presenting Darwin's evidence.

Ghislen, Michael T. *The Triumph of the Darwinian Method*. Berkeley: U of California P, 1969. "Mathematics" in the index (p. 21) defends Darwin's not using mathematics to make his geological arguments. He argues that something can be logical whether it is expressed in a language of formal logic like mathematics or not (p. 65). Again he attacks mathematical argument in the harder sciences (p. 127). Darwin uses geometry to describe changes in the physical shape of organisms (p. 170). Darwin quantifies different physical features in organisms to present precise morphological comparisons (p. 175). Discusses Darwin's hybridization experiments. It claims that his results were subjected to statistical analysis by Galton (though its not clear whether this was done before or after publication).

Gillespie, Neal C. *Charles Darwin and the Problem of Creation*. Chicago: U of Chicago P, 1979. Gillespie doesn't mention mathematics in the index. He doesn't discuss it with regards to Darwin's approach to his subject.

Hull, David. *Darwin and His Critics: The Reception of Darwin's Theory of Evolution by the Scientific Community*. Cambridge: Harvard UP, 1973. Hull has an entry for mathematics with 6 page references in the index. On p. 11 he comments how Darwin was no more confused about the differences between scientific laws, mathematical axioms, and metaphysical principles than most people in the period in which he lived. Hull claims that despite his misunderstanding of these differences Darwin understood the general distinction between deductive and inductive reasoning. On p. 14 Hull explains that Darwin was caught in the middle of a debate about the difference between, "the nature of mathematical axioms and their relation to their experience." On p. 20 Hull argues that mathematics for Aristotle was different from empirical science because it required fewer observations to reach a conclusion. On p. 32 Hull claims that Darwin's theory is not mathematical in form or in the type of reasoning. On p. 34 Hull claims that in the 1930s different scientist tried through genetics to create a mathematical science of evolutionary biology. This was not effective in creating verifiable outcomes, however. On p. 61 Hull argues that evolution did not exhibit mathematical regularity as did the laws in hard science.

Mayr, Ernst. *Charles Darwin and the Genesis of Modern Evolutionary Thought*. Cambridge: Harvard UP, 1991. Mayr doesn't mention mathematics in the index of his text. In chapter 5 (p. 48) Mayr notes that the concept of science in Darwin's time was completely dominated by mathematics and the physical sciences.

Appendix B

A Mathematical Table Created by Darwin Used in *Natural Selection*

Charles Darwin, *Charles Darwin's Natural Selection*, ed. R.C. Stauffer (Cambridge UP, 1975) 149.

VARIATION UNDER NATURE

TABLE I[1]

For particulars on the works here tabulated and on the few corrections made, see the Supplement to this Chapter.	The numerators in the columns give the number of species presenting varieties; the denominators the number of species in the larger and smaller genera: these fractions are all reduced to common denominators of a thousand for comparison, and are printed in larger type to catch the eye. The right hand rows of figures in the three columns, with decimals, show the average number of varieties which each varying species has,—thus the number 1.50 shows that each two varying species have on average between them three varieties.

	Larger Genera	*Smaller Genera* (including those with single species)	*Genera with a single species*
Great Britain. Bentham			
Great Britain: Babington—Larger Genera with 5 species and upwards, smaller with 4 species and downwards [Pencil note by C.D.: Write this column larger'.]	$\frac{101}{663} = \frac{152}{1000}$ 1.40	$\frac{89}{745} = \frac{119}{1000}$ 1.30 [Pencil note by C.D.: 'Write this larger'.]	$\frac{24}{255} = \frac{94}{1000}$ 1.50
Great Britain, Henslow—Larger Genera with 5 species and upwards, smaller with 4 species and downwards. The Varieties are divided into two groups, the less strongly marked, and those which have been ranked by some eminent Botanists as species. Lesser Vars:	$\frac{69}{560} = \frac{123}{1000}$ 1.55	$\frac{67}{692} = \frac{96}{1000}$ 1.40	
Stronger Vars:	$\frac{33}{560} = \frac{58}{1000}$ 1.33	$\frac{29}{692} = \frac{41}{1000}$ 1.20	
Great Britain—London Catalogue (1853) (see Supplement for nature of Varieties)—Larger Genera with 5 species and upwards, smaller with 4 species and downwards	$\frac{97}{616} = \frac{157}{1000}$ 1.35	$\frac{85}{642} = \frac{132}{1000}$ 1.27	
Great Britain—London Catalogue—forms ranked as species in this catalogue but which have been thought by some authors to be varieties. In this second line, larger genera with 5 species and upwards, smaller with 4, 3, and 2 species	$\frac{57}{559} = \frac{101}{1000}$	$\frac{14}{377} = \frac{37}{1000}$	

[1] [Darwin’s holograph draft for this table is in ULC vol. 16.1, fol. 167.]

Appendix C

Mathematical Tables Created by Darwin Used in *Natural Selection*

Charles Darwin, *Charles Darwin's Natural Selection* ed. R.C. Stauffer (Cambridge UP, 1975). 150-51.

VARIATION UNDER NATURE

Table 1 cont.

	Larger Genera	*Smaller Genera* (including those with single species)	*Genera with a single species*
Canary Islands, Webb & Berthelot—Larger Genera with 4 species and upwards, smaller with 3 and downwards.	$\frac{49}{421} = \frac{116}{1000}$	$\frac{42}{551} = \frac{76}{1000}$	
India (part of Flora) Hooker & Thomson—Larger Genera with 7 species and upwards, smaller with 6 species and downwards.	$\frac{21}{258} = \frac{81}{1000}$ 1.61	$\frac{13}{165} = \frac{78}{1000}$ 1.53	
Tierra del Fuego: Hooker—Larger Genera with 3 species and upwards, smaller with 2 species and downwards	$\frac{19}{177} = \frac{107}{1000}$ 1.57	$\frac{16}{163} = \frac{98}{1000}$ 1.37	
New Zealand: Hooker—Larger Genera with 4 species and upwards, smaller with 3 species and downwards	$\frac{52}{361} = \frac{149}{1000}$ 1.82	$\frac{37}{323} = \frac{114}{1000}$ 2.05	$\frac{15}{159} = \frac{94}{1000}$ 2.00
Insecta: Coleoptera Madeira: Wollaston—Larger Genera with 4 species and upwards, smaller with 3 species and downwards	$\frac{35}{225} = \frac{155}{1000}$ 1.71	$\frac{26}{257} = \frac{101}{1000}$ 1.34	
Sweden-Gyllenhal—Larger Genera with 11 species and upwards, smaller with 10 species and downwards	$\frac{512}{1344} = \frac{380}{1000}$ 1.85	$\frac{151}{485} = \frac{311}{1000}$ 1.43	$\frac{11}{43} = \frac{255}{1000}$ 1.54

that there were many great difficulties in the way. The subject is so highly important to us, as we shall see in a future chapter, that these difficulties must be discussed at tedious length; but it will be convenient first to give the tables./

A26/In Table 1, we have several of the best known local Floras, (some of which were selected for me by Dr. Hooker) with the species divided into two great groups, those in the larger & those in the smaller genera. On the extreme right hand we have the genera with only a single species, but these are likewise included amongst the smaller genera. Some of the smaller Floras have been selected simply from giving remote countries under different climates. I may premise that I have given every single Flora (&

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